A Macroscopic Description of Gauging — as a special Morita equivalence

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Outline

Gauging is usually done microscopically in field theories or lattice models to give a dual (gauged) theory.
We give a macroscopic/categorical understanding.
The following important data are needed for gauging:

- The symmetry assignment of the theory to be gauged
  a monoidal functor
- The precise way to perform gauging: which symmetry defects are to be condensed and how
  a module category or a condensation algebra of symmetry defects

The remaining things are just computing centers!

Outline

Symmetry assignment

The traditional way of assigning a symmetry G to a quantum system V is by specifying a group representation:

$$\rho : G \to GL(V) \subset \mathrm{End}(V).$$

- G is an abstract (indexing) group.
- $\mathrm{End}(V)$ is all linear operators on V.
- ρ is a group homomorphism a map that preserves multiplication.

It is important to separate the abstract multiplication rule from concrete operators. Even if G and V are the same, different ρ should be considered as different physical symmetries.

To proceed, we need to abandon the idea that

~~G acts externally on V.~~

Symmetry assignment

Generalized symmetry are formed by topological operators.

- If we collect all the possible topological operators in a physical system in n+1D, the corresponding mathematical description is a fusion n-category $\mathcal{A}$.
- The abstract (indexing) generalized symmetry is also a fusion n-category $\mathcal{T}$.

Definition

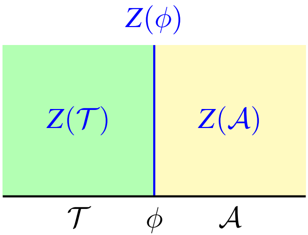
A symmetry assignment is a higher linear monoidal functor

$$\phi : \mathcal{T} \to \mathcal{A}.$$

Center functor

The center functor plays a central role in gauging.
A useful physical picture is to think center computes bulk.

- Center of a fusion n-category $\mathcal{C}$: $Z(\mathcal{C}) = \mathrm{Fun}_{\mathcal{C}|\mathcal{C}}(\mathcal{C}, \mathcal{C})$.
- Center of $\mathcal{C}$-$\mathcal{D}$-bimodule $\mathcal{M}$: $Z_{\mathcal{C}|\mathcal{D}}(\mathcal{M}) = \mathrm{Fun}_{\mathcal{C}|\mathcal{D}}(\mathcal{M}, \mathcal{M})$.
- Center of $\phi : \mathcal{T} \to \mathcal{A}$: $Z(\phi) := Z_{\mathcal{T}|\mathcal{A}}({}_\phi\mathcal{A})$, where ${}_\phi\mathcal{A}$ is the $\mathcal{T}$-$\mathcal{A}$-bimodule $\mathcal{A}$ with left action given by ϕ.



Center depicted as bulk.

Physical meaning of $Z(\phi)$

Informally speaking, $Z(\phi)$ is the centralizer or commutant of $\phi(\mathcal{T})$ in $\mathcal{A}$:

$$Z(\phi) \sim \{X \in \mathcal{A} | \, X \otimes \phi(s) \cong \phi(s) \otimes X, \forall s \in \mathcal{T}\}.$$

$Z(\phi)$ is the topological operators that are invariant under the action of $\phi(\mathcal{T})$, recall that a unitary symmetry acts on operators by conjugation $U_g X U_g^\dagger$, which may be called the charge/representation category.

Example: for the forgetful functor $\phi : n\mathbf{Vec}_G \to n\mathbf{Vec}$, $Z(\phi) = n\mathrm{Rep}G$.

Zipping

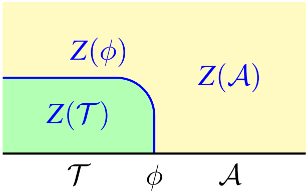
The symmetry assignment $\phi : \mathcal{T} \to \mathcal{A}$ is equivalent to the "sandwich" picture.

L. Kong and H. Zheng, The center functor is fully faithful, Advances in Mathematics 339 (2018) 749 [1507.00503]

L. Kong and H. Zheng, Categories of quantum liquids II, 2107.03858

D.S. Freed, G.W. Moore and C. Teleman, Topological symmetry in quantum field theory, 2209.07471.

One direction, we call Zipping:
Computing the center gives a sandwich $\mathcal{T} \underset{Z(\mathcal{T})}{\boxtimes} Z(\phi) \cong \mathcal{A}$.



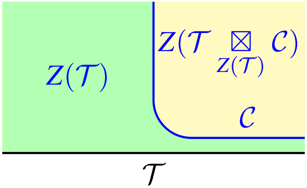
Unzipping

The other direction, we call Unzipping:
Sandwich $\mathcal{T} \underset{Z(\mathcal{T})}{\boxtimes} \mathcal{C} \cong \mathcal{A}$ implies a natural symmetry assignment:

$$\eta_{\mathcal{T},\mathcal{C}} : \mathcal{T} \to \mathcal{T} \underset{Z(\mathcal{T})}{\boxtimes} \mathcal{C}$$
$$s \mapsto s \underset{Z(\mathcal{T})}{\boxtimes} \mathbf{1}_{\mathcal{C}}$$

The center of this symmetry assignment is exactly $\mathcal{C}$,

$$\mathcal{C} \cong Z(\eta_{\mathcal{T},\mathcal{C}}).$$



Abstract gauging

The ways of gauging should be first specified for the abstract symmetry $\mathcal{T}$:

Definition

An abstract gauging of the abstract symmetry $\mathcal{T}$ is an indecomposable right $\mathcal{T}$-module $\mathcal{K}$.

Equivalently, an abstract gauging can be specified by an algebra K in $\mathcal{T}$ such that $\mathrm{LMod}_K(\mathcal{T}) \cong \mathcal{K}$. The symmetry defects contained in the algebra K are to be "summed over" or "condensed".

Abstract gauging

Definition

An abstract gauging of the abstract symmetry $\mathcal{T}$ is an indecomposable right $\mathcal{T}$-module $\mathcal{K}$.

Denote by $\mathcal{T}^\vee_\mathcal{K} := Z_{n\mathbf{Vec}|\mathcal{T}}(\mathcal{K}) = \mathrm{Fun}_{n\mathbf{Vec}|\mathcal{T}}(\mathcal{K},\mathcal{K})$ the center of $\mathcal{K}$, called the gauge symmetry or dual symmetry (of $\mathcal{T}$ with respect to $\mathcal{K}$).

$\mathcal{T}^\vee_\mathcal{K}$ is Morita equivalent to $\mathcal{T}$ by definition.

Abstract gauging

Definition

An abstract gauging of the abstract symmetry $\mathcal{T}$ is an indecomposable right $\mathcal{T}$-module $\mathcal{K}$.

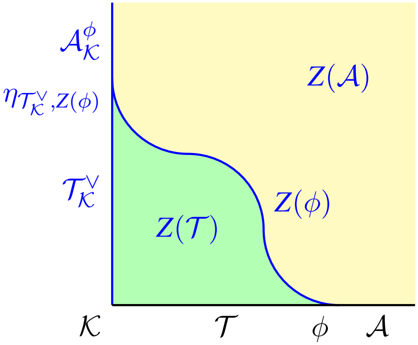
If $n\mathbf{Vec}$ is a $\mathcal{T}$-module, $\mathcal{T}$ is a local fusion n-category (free of 't Hooft anomaly) and choosing $\mathcal{K} = n\mathbf{Vec}$ corresponds to the ordinary gauging, i.e., "summing over" all possible symmetry defects.

In general, $\mathcal{K}$ can be larger and corresponds to a partial gauging where only a subset of symmetry defects, instead of all, are "summed over".

Gauging prescription

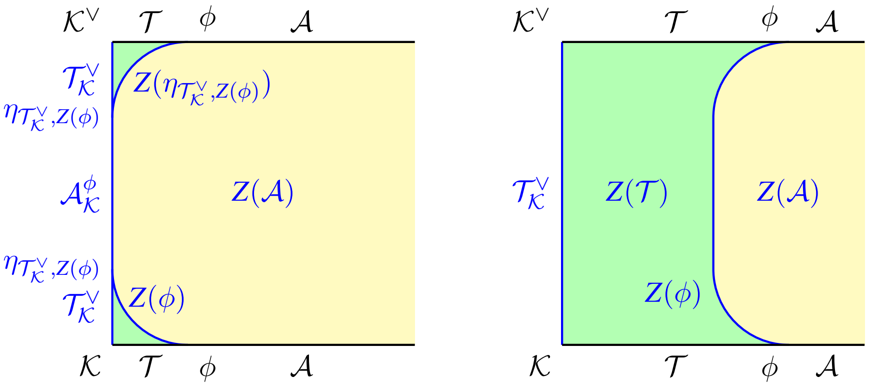
Definition

Given a theory $\mathcal{A}$, we need to specify its symmetry $\phi : \mathcal{T} \to \mathcal{A}$, and an abstract gauging $\mathcal{K}$ of $\mathcal{T}$, and then the gauged theory is $\mathcal{A}^{\phi}_{\mathcal{K}} := \mathcal{T}^{\vee}_{\mathcal{K}} \underset{Z(\mathcal{T})}{\boxtimes} Z(\phi) \cong Z_{n\mathbf{Vec}|\mathcal{A}}(\mathcal{K} \underset{\mathcal{T}}{\boxtimes} {}_{\phi}\mathcal{A})$.

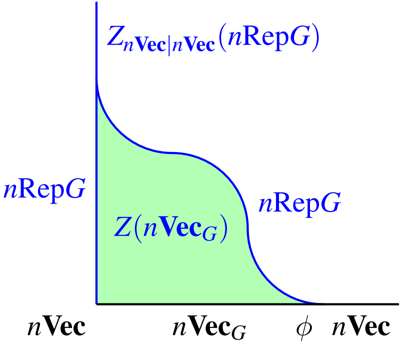


By Unzipping, $\mathcal{T}^{\vee}_{\mathcal{K}}$ is a natural symmetry of the gauged theory $\mathcal{A}^{\phi}_{\mathcal{K}}$, with the same charge category $Z(\phi)$ as the original theory.

Gauging is reversible (Morita equivalence)

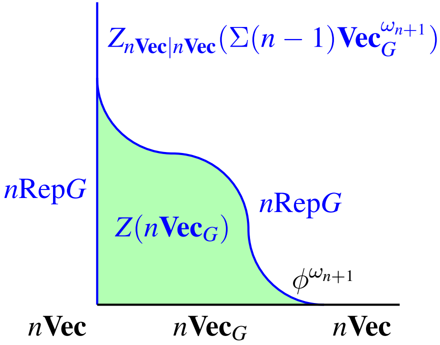


Ordinary gauge theory



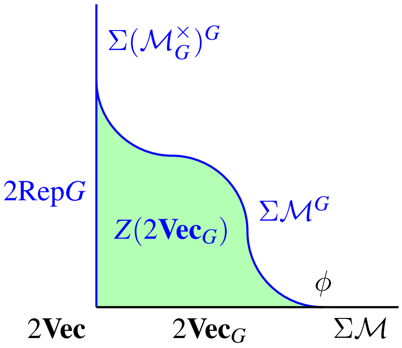
$\phi : n\mathbf{Vec}_G \to n\mathbf{Vec}$ is the forgetful functor. The gauged theory is the ordinary G-gauge theory.

Dijkgraaf-Witten gauge theory



$\phi^{\omega_{n+1}} : n\mathbf{Vec}_G \to n\mathbf{Vec}$ is twisted by an (n+1)-cocycle ω_{n+1}, describing a SPT phase. The gauged theory is the Dijkgraaf-Witten G-gauge theory.

Gauging 2+1D SET phases



Take a UMTC $\mathcal{M}$ with a G-action. It can be gauged iff there exists a G-crossed braided extension $\mathcal{M}_G^\times$, and in the 2-categorical language, iff there exists a monoidal functor $\phi : 2\mathbf{Vec}_G \to \Sigma\mathcal{M}$.

ENO arXiv:0909.3140, BBCW arXiv:1410.4540

$\mathcal{M}^G$, the equivariantization of $\mathcal{M}$, is a UMTC over $\mathrm{Rep}G$; $(\mathcal{M}_G^\times)^G$ is a minimal modular extension of $\mathcal{M}^G$.

LKW arXiv:1602.05936,1602.05946

Other Examples

- Partial gauging: take a subgroup $L \subset G$. The abstract gauging $\mathcal{K} = \mathrm{LMod}_{(n-1)\mathbf{Vec}_L}(n\mathbf{Vec}_G)$ is equivalent to partially gauging the subgroup L.
- Higher gauging: we allow $Z(\mathcal{A})$ to be nontrivial, in which case gauging $\mathcal{A}$ is a higher gauging inside a nontrivial bulk.
- "Lower" gauging: we can as well gauge the bulk theory $\Sigma Z(\mathcal{A})$. A typical example is gauging fermion number parity.

Conclusion and Outlook

- Symmetry assignment $\phi : \mathcal{T} \to \mathcal{A}$ is equivalent to sandwich $\mathcal{T} \underset{Z(\mathcal{T})}{\boxtimes} Z(\phi) \cong \mathcal{A}$.
- Abstract gauging $\mathcal{K}$ is a right $\mathcal{T}$-module.
- Gauged theory is the center of right $\mathcal{A}$-module $\mathcal{K} \underset{\mathcal{T}}{\boxtimes} {}_{\phi}\mathcal{A}$ and thus Morita equivalent to $\mathcal{A}$.
- Our framework covers all known variants of gauging, and may be used to discover unknown ones

Thanks for attention!